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Let $y=v(x+1)$; then the transformed equation is

$$\frac{d^2v}{dx^2}+\left\{\frac{2}{1+x}-\frac{9x^2+10x+1}{6x^3+20x^2-2x}\right\}\frac{dv}{dx}=0$$

$$\therefore\ \frac{dv}{dx}(1+x)^2 e-\int\frac{9x^2+10x+1}{6x^3+20x^2-2x}dx=A.$$

$$\therefore\ \frac{dv}{dx}=\frac{A}{(1+x)^2}e\int\frac{9x^2+10x+1}{6x^3+20x^2-2x}dx=\frac{3x^2+10x-1}{x^{\frac{1}{2}}(1+x)^2}A.$$

$$\therefore\quad V=\frac{6x^{\frac{3}{2}}-2x^{\frac{1}{2}}}{1+x}A+B.\qquad \text{But } y=v(x+1),\quad \because\ y=2x^{\frac{1}{2}}(3x-1)A+(1+x)B,$$

a complete solution, where A and B are arbitrary constants.

Particular solutions are $y=1+x$ and $y=2x^{\frac{1}{2}}(3x-1)$.

III. **Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio, and the PROPOSER.**

Assume $y=A_0x^m\ldots.(1)$; that the given equation becomes

$(6m^2-15m+9)x^{m+1}+(20m^2-30m+1)x^m-(2m^2-m)x^{m-1}=0\ldots.(2)$.

If $m'=m-1\ldots.(3)$, $m'+5=m\ldots.(4)$, $\kappa=1\ldots.(5)$.

Put, then, $y=\Sigma_0^\infty A_rx^{m+r}\ldots.(6)$; and the given equation becomes again

$$\Sigma_0^\infty\Big\{[6(m+r)(m+r-2)-9(m+r)+9]A_rx^{m+r+1}$$
$$+[20(m+r)(m+r-1)-10(m+r)+1]A_rx^{m+r}$$
$$-[2(m+r)(m+r-1)+(m+r)]A_rx^{m+r-1}=0\ldots.(7).$$

Putting for r, $r-2$, $r-1$ in the first two terms, (7) becomes

$$\Sigma_0^\infty\Big\{[6(m+r-1)(m+r-3)-9(m+r-2)+9]A_{r-2}$$
$$+[20(m+r-1)(m+r-2)-10(m+r-1)+1]A_{r-1}$$
$$-[2(m+r)(m+r-1)+(m+r)]A_r\Big\}x^{m+r-1}=0\ldots.(8).$$

Making the coefficient of A_r vanish after putting $r=0$, we have $m=0\ldots.(9)$, $m=\frac{1}{2}\ldots.(10)$.

Substituting $m=0$ in the first two terms, and writing $r+2$ for r, $(6r^2-15r+9)A_r+(20r^2+10r-9)A_{r+1}=0\ldots.(11)$. Putting $r=0$, $r=1$ in succession in (11), $A_1=A_0$, $A_2=0$; then one part of the integral is $y'=A_0(1+x)\ldots.(12)$. The like step for $m=\frac{1}{2}$ gives another part of the integral $y''=B_0x^{\frac{1}{2}}(1-3x)\ldots.(13)$.

The whole integral is $y=y'+y''=A_0(1+x)+B_0x^{\frac{1}{2}}(1-3x)\ldots.(14)$.

[This result is also correct since the constant $A'=my-2A$. This has proved a very interesting problem. Prof. Matz sent four different solutions.—Ed.]

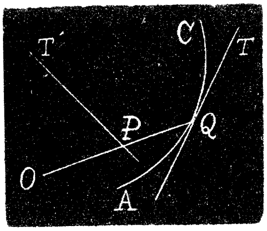
Also solved by *H. C. WHITAKER* and *C. E. WHITE.*

23. **Proposed by W. I. TAYLOR, Baldwin University, Berea, Ohio.**

From a point O situated in the plane of a plane curve, radii vectores are drawn to different points of the curve, and on each one a distance is laid off from O inversely proportional to the length of the radius vector: to determine the tangent at any point of the locus of the points thus obtained. [*Byerly's Diff. Calculus*, p. 177, *ex. 1.*]

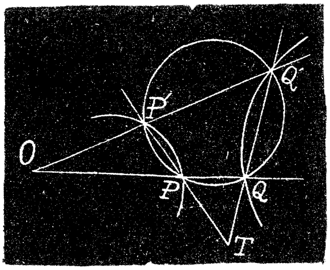
I. Solution by WILLIAM E. BYERLY, Ph. D., Professor of Mathematics in Harvard University, Cambridge, Massachusetts.

Let AC be an arc of the given curve and let P be the given point of the reciprocal curve at which a tangent is to be drawn. Draw the radius vector OP and extend it until it meets AC at Q. Draw the tangent QT at Q, and at P draw the line PT' making the angle OPT' equal to the supplement of OQT. Then will PT' be the required tangent. The proof of the correctness of this solution is obvious.



II Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

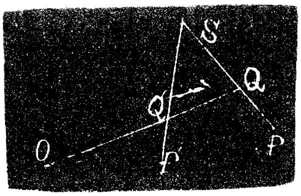
Let P and P' be two contiguous points on a curve, and Q and Q' the *inverse points*; then, since $OP\times OQ=OP'\times OQ'=k^2$, the points P, P', Q' and Q are concyclic; and, since the angles OPT and $OQ'T$ are therefore supplementary, it follows that, in the limit when P' ultimately coincides with P and Q' with Q, the tangents at P and Q make supplementary angles with OPQ.



If the Cartesian Co-ordinates of P be (x,y), and of Q (x',y'); then must $x=k^2x'/(x'^2+y'^2)$, and $y=k^2y'/(x'^2+y'^2)$. Consequently the inverse of the conic $ax^2+2bxy+cy^2=2y$, is the cubic $k^2(ax^2+2bxy+cy^2)=2y(x^2+y^2)$; also, the *inverses* of the lines $x=a$ and $y=b$, are respectively $x^2+y^2=k^2x/a$ and $x^2+y^2=k^2y/b$; and the *inverses* of the straight lines $3x+4y=5$ and $4x-3y=5$, with regard to the co-ordinate origin, are *orthogonally—intersecting* circles.

III. Solution by C. E. WHITE, Trafalgar, Indiana.

If Q' and P' be two points on the vectors OQ and OP, such that $OQ'.OQ=OP'.OP$, it follows that the points Q', Q, P, and P' be on a circle.



Now when P and Q coincide the lines QP and $Q'P'$ become the tangents to the inverse points P and P'. They, also, become the tangents to the circle at these points; hence, the $\triangle SPP'$ is isosceles. Hence whenever the tangent of P is known the tangent at P' may be drawn.

Again if X and Y be the co-ordinates of P' and x and y those of P, then (*Williamson's Calculus*, p. 224) $X=\dfrac{k^2x}{x^2+y^2}$ and $Y=\dfrac{k^2y}{x^2+y^2}$.

Whence $dX=k^2[(y^2-x^2)dx-2xydy]\div(x^2+y^2)^2$ and $dY=k^2[(x^2-y^2)dy-2xydx]\div(x^2+y^2)^2$.

$$\text{Hence } \frac{dX}{dY}=\frac{(x^2-y^2)dy-2xydx}{(y^2-x^2)dx-2xydy}.$$

Hence, if x' and y' be the co-ordinates of a point on the curve, the tangent at its inverse point may be

$$\text{written } \left(y-\frac{k^2y'}{x'^2+y'^2}\right)=\frac{(x'^2-y'^2)dy'-2x'y'dx'}{(y'^2-x'^2)dx'-2x'y'dy'}\left(x-\frac{k^2x'}{x'^2+y'^2}\right).$$

Also solved by *G. B. M. ZERR.*

PROBLEMS.

30. Proposed by E. W. NICHOLS, Professor of Mathematics in the Virginia Military Institute, Lexington, Virginia.

Given the cardioid $r=a\ (1-\cos\theta)$; find the area of its circumscribing square formed by tangents making angles of 45° with its axis.

31. Proposed by E. B. ESCOTT, Ann Arbor, Michigan.

Through a point O on the produced diameter AB of a simicircle draw a secant ORR', so that the quadrilateral $ABRR'$ inscribed in the semicircle shall be a maximum. Prove that in this case, the projection of RR' on AB is equal in length to the radius of the circle. [*Williamson's Diff. Calculus, 7th edition, p. 189, Ex, 25.*]

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MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

12. Proposed by Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

A horizontal table without weight is supported on three points A, B, and C. A weight W is laid upon the table, at a point G. If $AG=a$, $BG=b$, $CG=c$, $\angle AGB=\beta$, and $\angle AGC=\nu$, find the pressures upon A, B, and C.

I. Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland, and L. B. FRAKER, Weston, Ohio.

From the problem, we have $AB=\sqrt{(a^2+b^2-2ab\cos\beta)},=m$. Simi-